Answer Sheet to the Written Exam

Corporate Finance and Incentives

February 2017

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student should be able to:

Knowledge:

1. Understand, account for, define and identify the main methodologies, concepts and topics in Finance
2. Solve standard problems in Finance, partly using Excel
3. Criticize and discuss the main models in Finance, relating them to current issues in financial markets and corporate finance

Skills:

1. Manage the main topics and models in Finance
2. Organize material and analyze given problems, assessing standard models and results
3. Argue about financial topics, putting results into perspective, drawing on the relevant knowledge of the field

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems, and on given descriptions of situations in financial markets or corporations
2. Be prepared for more advanced models and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1 and 2, skills of type 1 and 2, competencies 1 and 2. Problem 4 emphasizes knowledge points 1 and 3, skills 1 and 3, an competency 1.

Some numerical calculations may differ slightly depending on the software used for computation, so a little slack is allowed when grading the answers.

Problem 1 (Asset Pricing 20%)

1) To solve $\varphi^T = d^T V$, note that $d^T = \varphi^T V^{-1}$ and use matrix inversion in Excel. The solution for d^T is $(0.228, 0.182, 0.302, 0.264)^T$.

2) Since all four entries in d are strictly positive, arbitrage is impossible. This is a consequence of Theorem 6.1.1 in the textbook by Ross.

3) From $\varphi^T = d^T V$, we already have $\varphi_i = \sum_{j=1}^{4} d_j V_{ji}$. If we normalize d so that its entries sum to one, we obtain $p^T = (0.233, 0.187, 0.309, 0.271)$. In this normalization, we have $d^T = 0.976 p^T$. We are done if we let $0.976 = 1/(1+r)$, i.e., let $r = (1/0.976) - 1 = 2.48\%$.

4) Let us suppose that arbitrage is still not possible. By the theorem, then this new asset also has $\varphi_5 = \sum_{j=1}^{4} d_j V_{j5}$ or $\varphi_5 = (1/(1+r)) \sum_{j=1}^{4} p_j V_{j5}$. Both expressions give the price $\varphi_5 = 4.396$.

Problem 2 (Debt 30%)

All amounts will be stated in million Kroner.

1) The corporate tax is 30% of 105 and the tax on equity holders is 20% of the remaining 70% of 105. All in all, the tax bill next year is 44% of 105. The present value of 105 is 100, so the total tax bill has present value 44. The equity holders get the residual which must then have present value $100 - 44 = 56$. There are of course other ways to compute these numbers, but the results should be the same.

2) On the left hand side of the equation is D which is the amount that creditors will pay today. On the right hand side is the present value of the cash flow that creditors receive next year. Partly, they will be repaid the amount D that they lent out, partly the will receive the interest payment $50 - D$, but this latter payment is subject to taxation on interest income. The equation is linear. It can be solved for

$$D = \frac{(1-\tau_i)\,50}{1 + r + \tau_i} = 46.15,$$

and then the interest is the remaining $50 - D = 3.85$.

3) The corporate tax will now be 30% of $105 - 3.85$. Thus, the cash that can be given to shareholders next year is $105 - 50 - 0.3\,(105 - 3.85) = 24.65$. The personal tax on this is 20% of this, i.e., 4.93.

4) The future tax bill contains the corporate tax, $30\%\,(105 - 3.85) = 30.34$, the tax on equity income from 3), 4.93, and the tax on interest income, $40\%\,(3.85) = 1.54$. The present value of this is $(30.34 + 4.93 + 1.54)\,/\,(1.05) = 35.06$. Today's tax paid by shareholders is $20\%46.15 = 9.23$. The present value of the total tax bill is thus $35.06 + 9.23 = 44.29$.

5) By assumption, in the perfect capital market, the net present value accruing to creditors is zero. Comparing results in 1) and 4), the present value of the tax bill is higher with

debt. Since the asset provides the same cash flow, we can conclude that shareholders lose from the releveraging. We could also more directly collect terms, to compute the new value of equity. From 3), shareholders get 24.65 next year, before tax. Today, they get D before tax. They can keep 80% of this, so equity is worth $80\%\,(24.65/1.05) + 80\%\,(46.15) = 55.71$.

Problem 3 (Real Options 25%)

1) Suppose the state is high. $I_1 > 0$ results in final payoff $200 - I_0 - I_1$, while investment 0 results in $100 - I_0$. The best choice is to invest I_1 if $I_1 < 100$, with indifference where $I_1 = 100$. Suppose the state is low. Since the payoff is $50 - I_0$ less the amount invested at this stage, the best choice is to invest 0.

2) Suppose the state is high. $I_1 > 0$ results in final payoff $40 - I_1$, while investment 0 results in 0. The best choice is to invest I_1 if $I_1 < 40$, with indifference where $I_1 = 40$. Suppose the state is low. The situation is the same, so the best choice is to invest I_1 if $I_1 < 40$, with indifference where $I_1 = 40$.

3) Case 1, suppose $I_1 \leq 40$. The choice of $I_0 > 0$ will, according to 1), optimally be followed by investment of I_1 only in the high state. The payoff will then be $40\%\,(200 - I_0 - I_1) + 60\%\,(50 - I_0) = 110 - I_0 - 40\% I_1$. The choice of initial investment 0 will, according to 2), optimally be followed by investment of I_1 in both states. The payoff will be $40 - I_1$. The best choice is to invest $I_0 > 0$ when $I_0 < 70 + 60\% I_1$, with indifference at $I_0 = 70 + 60\% I_1$.

Case 2, suppose $40 < I_1 \leq 100$. The choice of $I_0 > 0$ will, as in case 1, result in payoff $110 - I_0 - 40\% I_1$. The choice of initial investment 0 will, according to 2), be followed by investment of 0 in both states. The payoff will be 0. The best choice is to invest $I_0 > 0$ when $I_0 < 110 - 40\% I_1$, with indifference at $I_0 = 110 - 40\% I_1$.

Case 3, suppose $100 < I_1$. The choice of $I_0 > 0$ will, by 1), be followed by 0 in both states. The payoff will be $40\%\,(100 - I_0) + 60\%\,(50 - I_0) = 70 - I_0$. The choice of initial investment 0 will, as in case 2, result in payoff 0. The best choice is to invest $I_0 > 0$ when $I_0 < 70$, with indifference at $I_0 = 70$.

Problem 4 (Various Themes 25%)

1) See chapter 6 from Grinblatt and Titman. The risk premium satisfies $\lambda_i = \sum_{k=1}^{K} \beta_{ik} \lambda_k$.

2) See Section 20.3 from Berk and DeMarzo.

3) Section 31.3 in Berk and DeMarzo discusses taxation and the repatriation of earnings, including the possibility of such a tax holiday. While 31.4 discusses the implication of internationally segmented capital markets, note that chapter 16 discusses potential distress implications of having insufficient capital — from this point of view, it might be reasonable to expect some U.S. to increase investment levels in the U.S., once they have a higher capitalization there.